Investigation of the Effect of Induced Spatial Incoherence on Laser/Plasma Instabilities

1990

A Proposal to the Laser-Plasma Branch
of the U. S. Naval Research Laboratory

by

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Introduction and Summary of Accomplishments to Date

Theoretical and experimental studies have shown [1-5] that Induced Spatial Incoherence (ISI) can have a large effect on plasma instabilities. In particular, it is possible that ISI can suppress or substantially raise the threshold for filamentation instabilities. Such an effect would have important implications for inertial confinement. The present principal investigator, in collaboration with Prof. Allen Boozer (also of William and Mary), has developed a novel approach to the study of self-consistent wave-plasma interaction problems such as ISI driven filamentation [6]. The central idea is to use the geometric optics approximation to describe the light propagation and then self-consistently couple this to the plasma. This can be done either using an action principle [6] or via an energy-momentum balance calculation starting from Maxwell's equations [7]. The latter approach is more general since it allows dissipative effects (such as thermal transport) to be included.

Associated with the study of ISI via the wave-kinetic formalism, we are also engaged in an investigation to better understand the limitations of the wave-kinetic formalism in general. This work is being carried out in conjunction with Prof. A. N. Kaufman at UC Berkeley and has resulted in the discovery that linear mode conversion (previously thought outside the domain of phenomena describable by wave-kinetic theories) can be treated as a quasiparticle process if the incoming waves are incoherent [8]. This led to the further discovery that incoherent linear mode conversion is dissipative in the sense that it generates wave entropy even though it conserves energy, momentum and action. As a side comment, we mention that the ponderomotive generation of sound waves by laser driving is effectively dissipative also, even though the process naively appears conservative (i.e. even when the wave propagation is nondissipative a wave pulse can leave behind a wake which contains energy).

Since the mode conversion work is now in print, we will focus mainly on the filamentation study and then discuss directions for future work.

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The Effects of ISI on Filamentation

First we consider the case without dissipation. In this situation, filamentation, if it occurs, will be driven by ponderomotive effects. The self-consistent wave-kinetic equations in this case are:

$$\frac{\partial f}{\partial t} + v_g \cdot \nabla f - \nabla\omega \cdot \frac{\partial f}{\partial k} = 0, \tag{1}$$

which governs the radiation transport and

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0, \tag{2a}$$

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla\left[p_F + \int \frac{d^3k\, f}{\omega}\right] \tag{2b}$$

which govern the fluid motion. In the above equations:

$f = f(x,k;t)$ is the photon number density, or equivalently the action density;
$\omega = \omega(x,k;t) = (k^2c^2 + \omega_{pe}^2(x,t))^{1/2}$ is the dispersion relation for electromagnetic waves;
ω_{pe} is the local plasma frequency;
$v_g = \partial\omega/\partial k$ is the group velocity;
ρ = the mass density;
u = the Eulerian fluid velocity;
p_F = the fluid thermal pressure.

As we showed in [6] it is possible to recover the standard filamentation growth rate from eqs.(1)-(2) by linearizing about a uniform equilibrium with no fluid flow. To treat ISI it is simply necessary to generalize that earlier calculation to allow the leading order laser field to be non-uniform and time dependent. Specifically, we write:

$$\rho(x,t) = \rho_0 + \varepsilon\rho_1(x,t)\,; \qquad u(x,t) = \varepsilon w(x,t)\,; \qquad f(x,k,t) = \varepsilon f_0(x,k,t) + \varepsilon^2 f_1(x,k,t)\,.$$

A straightforward perturbation calculation then leads to the following equations:

$$\frac{\partial f_0}{\partial t} + c^2 \frac{\mathbf{k}}{\omega_0} \cdot \nabla f_0 = 0;$$

(3a)

$$\frac{\partial f_1}{\partial t} + c^2 \frac{\mathbf{k}}{\omega_0} \cdot \nabla f_1 = \frac{\omega_{pe}^2}{2\omega_0}\left[\frac{c^2}{\omega_0^2}\rho_1 \mathbf{k} \cdot \nabla f_0 + \nabla \rho_1 \cdot \frac{\partial f_0}{\partial \mathbf{k}}\right];$$

(3b)

$$\frac{\partial^2 \rho_1}{\partial t^2} - C_s^2 \nabla^2 \rho_1 = \frac{\omega_{pe}^2}{2\rho_0} \nabla^2\left[\int \frac{d^3k}{\omega_0}(f_0 + \varepsilon f_1)\right].$$

(3c)

(In eq.(3c) c is the speed of light in vacuum and C_S the sound speed.)

We are in the process of developing a 2-D code to solve eqs.(3) in a manner appropriate to the study of ISI drivers. We solve the problem in the x-z plane, where x is the direction transverse to the laser beam, and z the predominant direction of propagation of the beam (see fig.(1), page 8, for the geometry). The laser beam is assumed to impinge on the plasma at z=0 and to propagate into the z>0 region. We assume the system is periodic in the x-direction. Thus, we solve eqs.(3) subject to the following initial and boundary conditions:

Initial conditions:

$\rho_1(x,0) = 0;$ $\qquad$ $f_0(x,k;0) = g(x,k);$ $\qquad$ $f_1(x,k,0) = 0.$

where the function g(x,k) is chosen to model an ISI beam which has been turned on for a long period of time. The spatial dependence of g(x,k) is determined by the statistics of the ISI beam, and it is strongly peaked about the carrier wavenumber $\mathbf{k}_0 = k_0\mathbf{z}$.

We can, of course, choose more general initial conditions for the density (e.g. to model sound turbulence driven by the ISI beam which is already on at the start of the simulation).

Boundary Conditions:

The system is periodic in x. In the z-direction the photons enter the system at z=0 and leave at $z=L_z$ (which is chosen to be several filamentation growth lengths). We are free to

specify the time dependent input photon distribution at z=0,

$$f_0(x, z = 0, \mathbf{k}; t) = h(x, \mathbf{k}; t)$$

and this is where ISI truly differs from other optical smoothing techniques. The input distribution function, h, for ISI drivers, varies on a time scale set by the coherence time of the laser ($\tau_c \approx 1/\Delta\omega \approx 10^{-12}$ sec). This is short compared to the transverse sound time ($\tau_s \approx L_x/C_s$). We choose the box size in the x-direction to be large enough so that there will be several filaments for the non-ISI case (i.e. a uniform driver). For 0.25 µm laser light at a power level of 10^{14} W-cm^2 this fixes L_x to be about 5×10^{-3} cm. For a 10 keV CH plasma this gives a sound time of $\tau_s \approx 10^{-9} - 10^{-10}$ sec $\approx 100 - 1{,}000\ \tau_c$.

Averaging:

There are two basic approaches for dealing with systems such as this which have such widely separated timescales: 1] examine model (nonphysical) situations where the timescales are chosen not to be too disparate. This allows the numerical solution of the full problem without further approximation and my give insight into the asymptotic limit where the physics is closer to reality. 2] Perform a multiscale expansion of some kind which allows the fast motion to be dealt with efficiently and then time average these results to drive the time averaged sound wave equation. So far we have focussed on the second approach, but the first may also give useful insight. More work needs to be done along these lines.

We are led to the following averaging scheme: both the laser distribution function, f, and the density, ρ, are split into fast and slow parts

$$f = f' + \langle f \rangle \qquad \rho = \rho' + \langle \rho \rangle \tag{3}$$

where the brackets indicate an average over timescales long compared to τ_c but short compared to τ_s and the primes indicate $\langle f' \rangle = \langle \rho' \rangle = 0$.

Eqs.(3a) and (3b) are followed on the fast time scale. The evolution of the density perturbation, eq.(3c) is treated separately as follows. The fast density response, ρ_1', follows the ISI driver and a simple estimate leads to the conclusion that it obeys:

$$\frac{\partial^2 \rho_1'}{\partial t^2} = \frac{\omega_{pe}^2}{2\rho_0} \nabla^2 \int \frac{d^3k}{\omega_0} f_0' . \tag{4}$$

Time averaging eq.(3c) leads, finally, to the equation for sound waves driven by ISI ponderomotive effects:

$$\frac{\partial^2 < \rho_1 >}{\partial t^2} - C_s^2 \nabla^2 < \rho_1 > = \frac{\omega_{pe}^2}{2\rho_0} \nabla^2 \left[\int \frac{d^3 k}{\omega_0} (< f_0 > + \varepsilon < f_1 >) \right]. \tag{5}$$

Eqs.(3)-(5) may be solved as follows: the ISI input gives f_0. This is computed over some number of coherence times, but on time scales short compared to the sound time. The unperturbed photon distribution, f_0, is separated into fast and slow part via Fourier decomposition. The fast part of the density response is computed using eq.(4). The perturbed photon distribution function is computed using eq.(3b) and then separated into fast and slow parts as well. The slow density response is now computed using eq.(5) to update the plasma density. This process is then repeated for some reasonable number of sound crossing times (10 -20) to insure that the system has had time to develop filaments if it is unstable.

The principal investigator is working in collaboration with a student, A. J. Neil, to develop the necessary code to study the effects of ISI on filamentation using this approach. The results to date are as follows: numerous finite difference schemes have been investigated, many appear to be numerically unstable. This has led us to try a hybrid Fourier-finite difference scheme similar to that employed by A. J. Schmitt [4]. Namely: we Fourier analyze in the x-direction (assumed periodic) and do finite differencing in z. This work is still in progress.

Analytic Approaches

Aside from the major issue of numerically studying the effects of ISI on filamentation, there are some possible analytical avenues which may lead to some insight. It is clear from the form of eqs.(3)-(5) above that filaments can only form if correlations develop between the perturbed photon distribution and the density. In the uniform case filaments occur because of the coherent focussing of rays by transverse density perturbations. The major new element in the ISI case is the appearance of the unperturbed distribution function, f_0, in the sound wave equation (eq.(3c)). If ISI suppresses filamentation then it is most likely the complexity of this f_0 driving which prevents the coherence between ρ_1 and f_1 needed to form filaments.

Consider now a group of photons from a single echelon launched over a time τ_c. This group has a transverse spatial width of W (determined by the size of the echelon) and

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Consider now a group of photons from a single echelon launched over a time τ_c. This group has a transverse spatial width of W (determined by the size of the echelon) and

a length $L \approx c\tau_c$.This group propagates through the plasma at about the speed of light and generates a sound wake field behind it. If filaments are to form this wake field must interact coherently with the next group of photons which comes through (at a time on the order of τ_c later). Otherwise their effects will not add coherently. Thus a detailed study of the sound field emitted by such photon groups would be very useful.

Proposed Research

We propose to continue the development of a 2-D numerical code to study the effects of ISI on filamentation instabilities. This will incorporate the self-consistent wave-kinetic model described earlier.

In tandem with the numerical work, we also propose to examine the nature of the sound wake fields generated by photon groups. This is motivated by the fact that -- if filaments are to form in the ISI case -- density disturbances generated by many different photon groups must add coherently to produce a substantial effect on the perturbed photon distribution. If the ISI driving prevents this coherent interaction then it should help to suppress filamentation.

We also propose to continue our development of the basic theory of wave-kinetics, in particular focussing on generalizations of our recent mode conversion work to higher order processes such as two plasmon decay.

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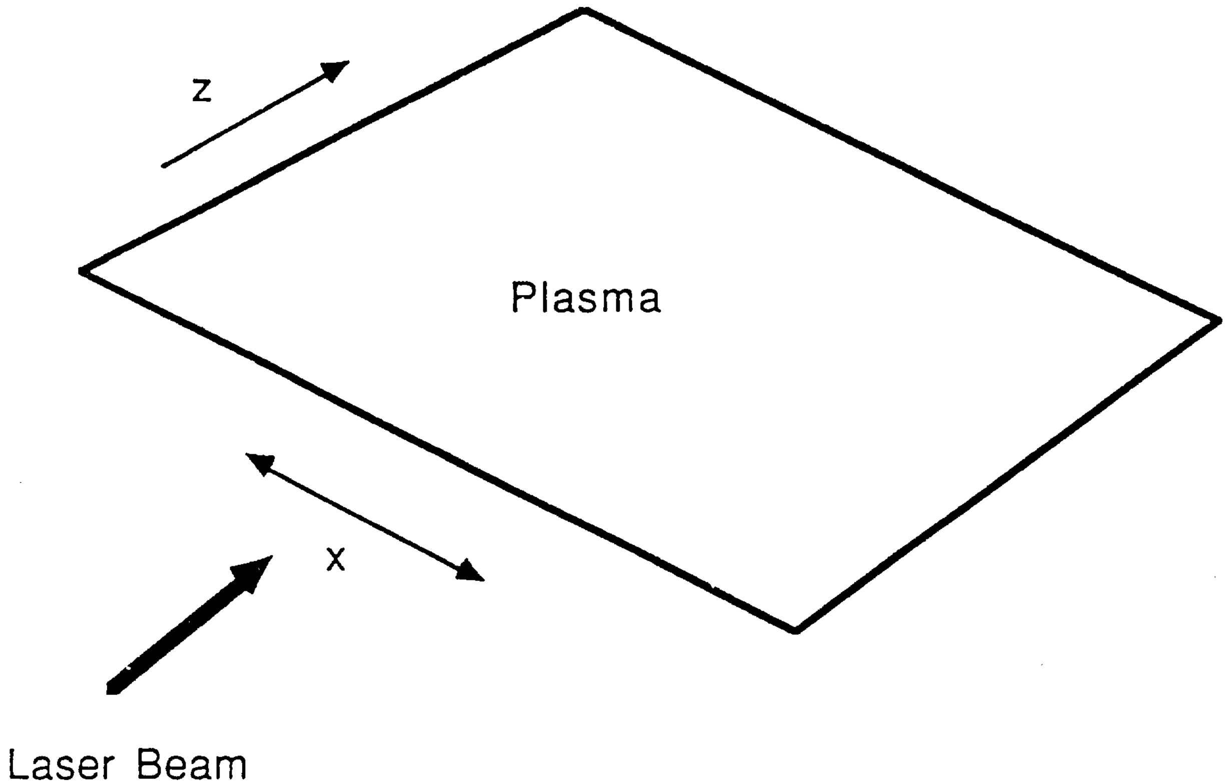

Fig.(1)